The Case for Decision Trees in Partition Actions

Roger J. Brown, PhD[1]

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Dr. Brown's extensive resume includes forty years' experience negotiating investment real estate sales, exchanges and leases; originating and servicing mortgage loans; fiduciary responsibilities of a general partner; expert witness in civil litigation; writing and teaching in the field of real estate finance, appraisal, taxation, analysis, and management; general manager of a firm established in 1967 that has had as many as 100 employees; and supervision of all aspects of finance, legal, and accounting administration for investment portfolios.

I. INTRODUCTION

Why is it that machines learn and people do not? This provocative, if argumentative, question is at the heart of this article. In privately held real estate investments, a problem arises when parties cannot agree on the management or disposition of a property owned in common. Differences of opinion on the future path for the property can arise due to the passage of time or some other event, such as the illness or death of a partner.[2]

II. THE DECISION TREE APPROACH

This article proposes a decision tree to analyze this difficulty and presents an automated tool to implement it. A fixture of operations research, the decision tree has been in use for many years.[3] Today, it forms the basis of widely used algorithms employed by machine learning. The reason each person on a plane has paid a different price for his or her ticket is because a decision tree optimized air carrier revenue.[4] Yet, people are often unwilling to use structured analysis to reach efficient solutions, bringing us back to the question asked in Section I above.

As a hypothetical, we will assume two owners, one being our client and the other being that person's co-owner or partner. Their respective percentage ownership shares are unimportant. Generally, absent an agreement, the idea of "partnership democracy" does not govern decisions for this venture.[5] Either the client or the partner could own five percent or ninety-five percent of the property, but in every case each must agree completely for anything to be done. This requirement for mutual agreement for every action is a difficult standard to meet, especially over time as facts on the ground change.

When parties do not agree on a given action and resort to a court for a solution, the court listens patiently and encourages the parties to reach agreement. However, if no solution is found, the court takes the rather dramatic and extreme path of supervising a sale and dividing the sale proceeds between the partners according to their respective ownership shares. Essentially, this division is a divorce in which the parties need not be married (although they might be). In many cases, the court has no way to divide property in kind (for example, a court could not evenly partition a five-unit apartment building between two partners), but once the property is liquidated to a third party, the court holds cash which can then be evenly divided. The court is not particularly good at this division, however, because the court's interest is the proper administration of the law rather than the economics of the resolution.

Eventually, the inefficiency of the court-supervised sale becomes apparent. In most cases, parties in dispute become educated by their respective attorneys, sometimes at great expense and often at different points in time. Recognizing that each party will lose money if they do not come to an agreement, the parties look for a non-judicial solution. Everything about this process is compounded in a case where, for example, one of the owners dies and his wife and five grown children replace him as owners, each with different spouses, children, goals, objectives, management capacity, understanding, time, and ability.

A. The Partnership Dissolution Problem as an Illustration

For the example in this Section II.A below, assume one party ("Client") wants to enter into a buy-sell agreement to resolve in advance all of the issues attendant with the demise of either party.[6] Assume the other party ("Partner") is unwilling. The narrative proceeds as if a mediator is retained to intervene. The decision tree takes the parties down various paths, each with a set of probabilities and end-of-game payoffs. Before introducing a set of fully elaborated examples, it is useful to describe the basic workings of a decision tree.[7]

Although decision trees can be more complex, the simplest involve a series of binary choices, occurring in order. Each pair of choices is mutually exclusive (taking one means you cannot take the other) and collectively exhaustive (only two choices exist and one must be taken). In Figure 1 below, the simplest decision tree, the starting point is today and the two choices are not very controversial.

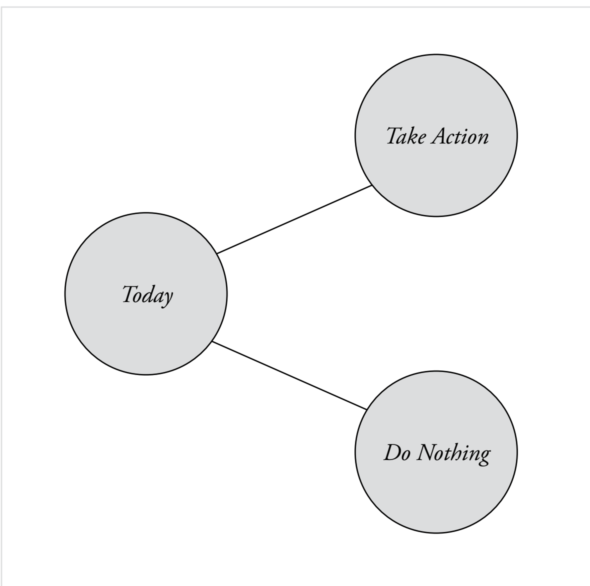

Figure 1

The simplest decision and its tree

B. Payoffs and Probabilities

Rational choice is based on seeking one's best interest. This necessitates a value scale so that one may distinguish between better or worse outcomes. Developing a stylistic example, however unrealistic at the outset, will assist here. Particular assumptions may not initially make sense; however, the heuristics demand proceeding in small steps. With these admonitions in mind, assume that each step involves a probability that some step will actually be taken. That probability appears adjacent to the line connecting the steps in Figure 2 below.[8] For now, assume that today there is an equal probability that one will do something or nothing so the probability for each is 0.5 or 50% as shown in Figure 2 below.

Due to the mutually exclusive and collectively exhaustive properties discussed in Section II.A above, at each node one need only know a single probability "p." Since the other choice must be taken if the first is not, its probability must be 1-p. So, for example, if one is told the person involved is ambitious, it is reasonable to assume that there is a higher probability, say 0.7 or 70%, that he or she will take action. This assumption automatically leaves the remaining choice, to do nothing, at a .3 or 30% probability, because both probabilities must equal one.

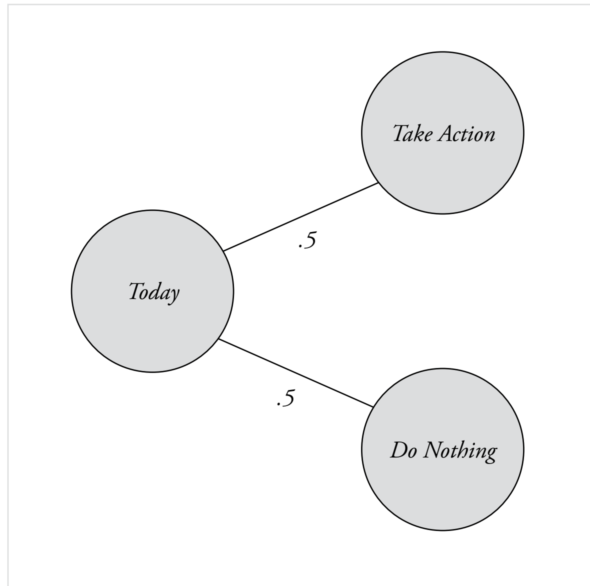

Figure 2

Probabilities added to the simplest decision

A second crucial assumption is that choices have consequences, which are referred to as "payoffs" and denoted with a number to the right of each action in Figure 3 below. While it is intuitively reasonable that choices have consequences, the devil is in the details when placing a value on a particular choice. This task is so difficult that it should not be addressed for the moment, but will be tackled below in Section II.C. Rather, simply observe that the numerical methodology is ordinal, not cardinal. In other words, the process requires only a ranking of numbers. Thus, whatever value is placed on a particular choice, the number tells us only how that choice compares to others. The values are not dollars; in fact, they have no units. It states that a payoff of ten is better than a payoff of nine. In Figure 3 below, a zero is arbitrarily added to each of the foregoing numbers to indicate that to "Take Action" has a payoff of 100 and that to "Do Nothing" has a payoff of 90.[9]

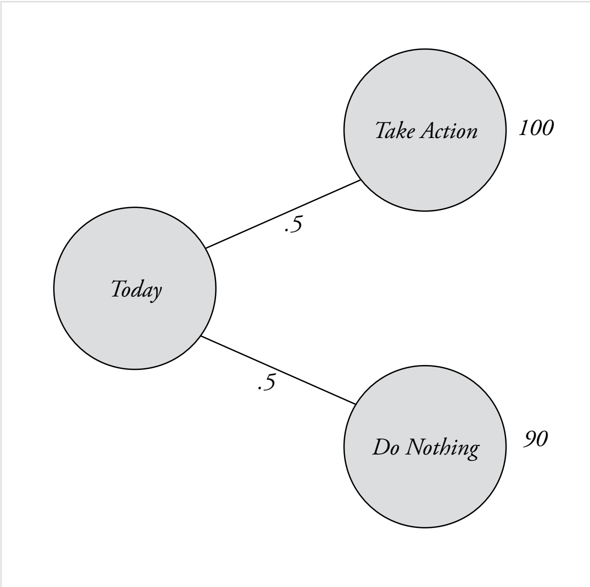

Figure 3

Values added to the simplest decision

C. Computing Expected Result

One more maneuver is necessary to round out the story. That maneuver is the computation of "expected result."[10] The mechanics of computing expected result, working from right to left, involve no more than multiplying the payoffs times their respective probabilities and then adding those products together. Thus, an example so far: expected result = (.5 x 100) + (.5 x 90) = 50 + 45 = 95. Such expected result appears in Figure 4 below.

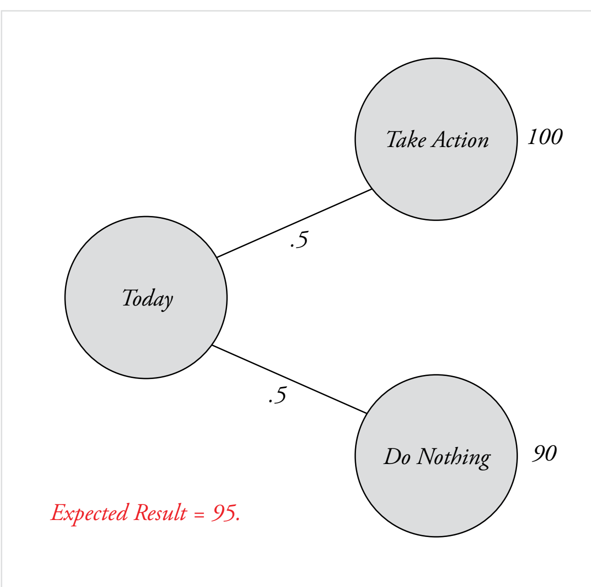

Figure 4

Computing the expected result

While different choices lead to different payoffs, they may also lead to additional choices. Thus, in Figure 5 below, matters are complicated by introducing some realism. While doing nothing is always an option, it may be that such a choice leaves nature to force particular outcomes, whether in the present or later in time.

Imagine two partners with differing age and health. Eventually, both partners will die, but one of them is likely to die first.[11] Consequently, the survivor will continue on without the other partner, perhaps dealing with heirs who do not understand the business the partners have jointly built and operated. Finally, if the age difference is great, the probabilities are not equal, as mortality tables indicate that the older partner may be expected to die first.[12]

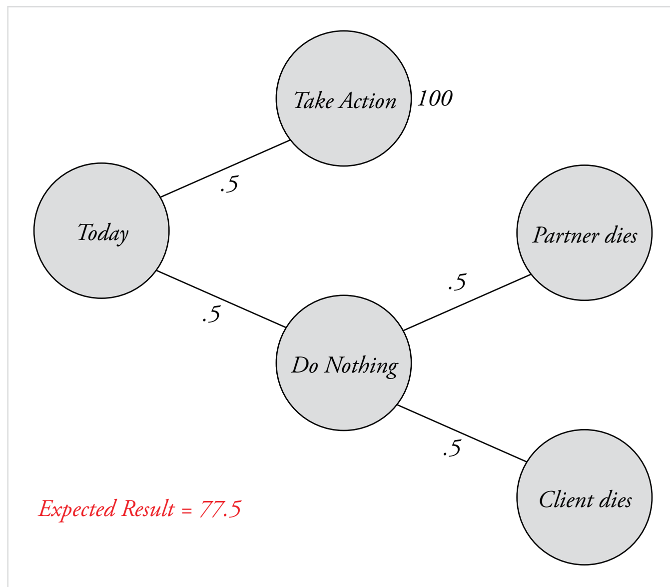

Figure 5

The simplest decision with added choices

The expected result in Figure 5 above is slightly more complex. Take the easiest number first. Beginning at the far right with the payoffs times the probabilities, compute each step separately. First, the "Do Nothing" node is: 60 x 0.5 + 50 x 0.5 = 30 + 25 = 55. In turn, that value is multiplied by its probability and added to the same computation for the "Take Action" node: 0.5 x 55 +.5 x 100 = 27.5 + 50 = 77.5.

As for the choice of payoff values, subjectivity is involved. So, if this example is being viewed from a client's standpoint, it is reasonable to believe that he or she would put the "payoff" of his or her own death below that of the partner's death.[13] Continuing the example, Figure 6 below provides two outcomes for the "Take Action" node: to negotiate or litigate. Each of these outcomes is given somewhat arbitrary values to permit the illustration to move forward.

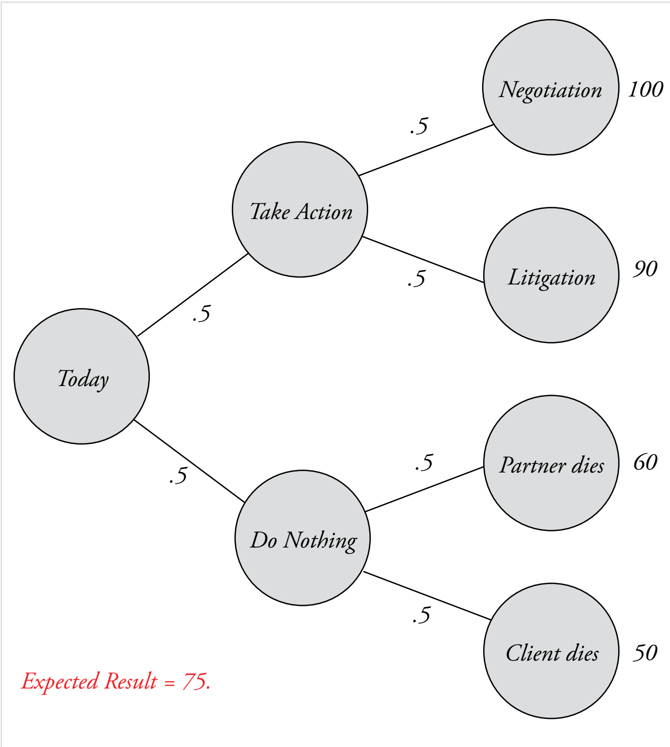

Figure 6

A more realistic tree

To round out the example, add choices arising from the "Negotiation" and "Litigation" alternatives; for the moment keep probabilities even and add arbitrary payoff values. The expected result in Figure 7 below is calculated in the same fashion as set forth above in this Section II.C.

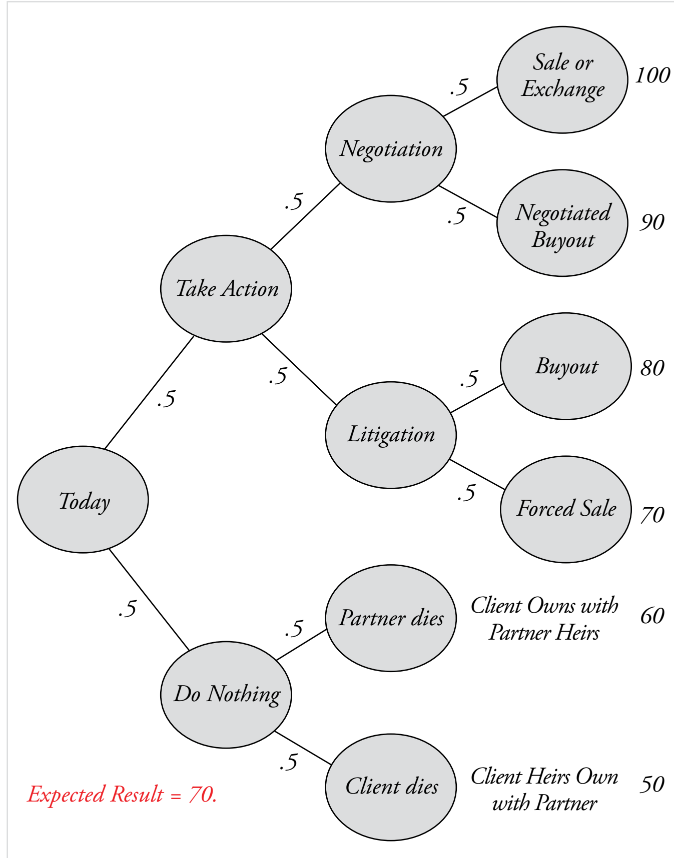

Figure 7

A fully elaborated tree with fixed values

D. Permutations of the Form

In the relatively simple example developed in Figures 1 through 7 shown above in this Section II, there are six payoffs and ten probabilities. The example can become more realistic and more complex by adding more nodes. For instance, assume that each of the conclusions in the top four (far right) outcomes may be at market or below market as buyers and brokers swarm properties that are the subject of ownership controversy.

Rather than expanding the overall structure of this example, note that its expected result changes under different assumptions. The calculation can be automated to relieve the tedium. Figure 8 below is a static image of an interactive graphic which provides the user with several alternatives and automatically calculates expected result under the chosen conditions.[14]

The example developed in Figures 1 through 7 shown above in this Section II assumed probabilities and payoffs were all fixed at the values given. That is the default for the "Fixed" button shown in Figure 8 below. At the other extreme, assume that all of those values are random. Selecting the "Random" button shown in Figure 8 below (and repeatedly using the "Re-randomize" button that appears when "Random" is selected),[15] values and probabilities are chosen Monte Carlo style[16] and a somewhat chaotic expected result is produced thereby.

In between the two extremes, "Fixed" and "Random," is the usual middle ground that culminates in partnership dissolution. In Section II.C above, the distinction was made between two deaths, making it easy to agree that one's own death is less desirable than someone else's.[17] Now, assume that there are three broad classes of personality types that characterize partners.[18] One is "Engaged" where the partners share authority and responsibilities equally. The other two are "Aggressive" and "Passive," each having its common meaning and presumed to apply to individual partners to different degrees. These three types are shown under the "Approach" button in Figure 8 below. Each type comes with a different set of payoffs and probabilities that, while still arbitrary, are less arbitrary than the "Fixed" case and less mechanistic than the "Random" case.[19] Without dwelling too much on the specifics, note the expected result for each of these three cases. Deliberately, the expected result declines as each individual custom choice is made in descending order.[20] This outcome suggests that "Engaged" provides the best outcome, "Passive" the worst, and "Aggressive" falls in the middle.

There is endless argument about whether these three classifications are useful, realistic, or in the proper order. Indeed, a persuasive case can be made that there should be thirty or three hundred rather than three classes. It is reasonable to argue that a world of seven billion people produces the same number of unique classes. As users of Facebook continue to increase, Facebook's algorithms have the opportunity to more accurately sort their users out. But, at the moment, this piece is concerned with a specific piece of real property and a small number of owners. It is beyond the scope of this article to explore the limits of big data.

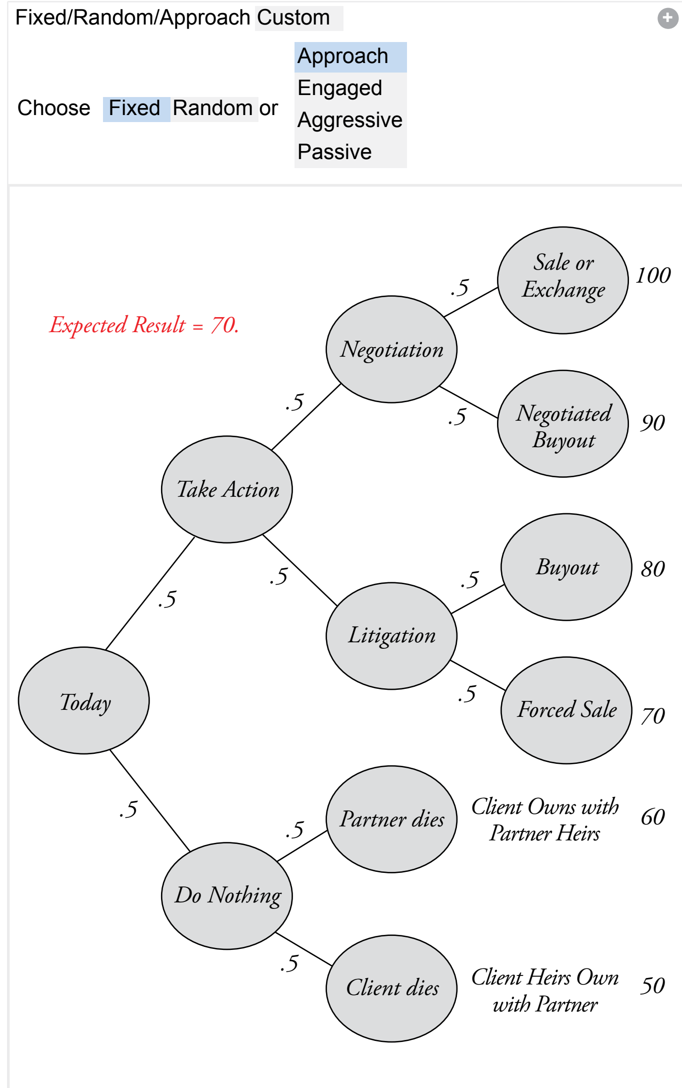

Figure 8

An automated approach to a decision tree

E. Client Participation

The three personalities described in Section II.D above may be a useful refinement over the fixed example portrayed in Figures 1 through 7 in Section II above. Imagine being a mediator seeking resolution between disagreeing partners. For this thought experiment, assume the mediator arrives with a story to tell, decision trees as the proposed methodology, and several copies of the graphic shown in Figure 9 below with all of the payoffs and probabilities blank. The mediator explains the theory, the method of calculation, and how the blanks are to be completed by each party, asking each party to assign their own probabilities and outcome values, with the calculation of the expected result to be determined at a later date.

In the "Fixed" example shown in Figure 8 in Section II.D above, notice that the outcomes at the top of the decision tree have higher values, reflecting the notion that dispassionate market solutions negotiated rationally by calm participants produce better results than outcomes imposed by third parties. But even the sub-optimal outcomes imposed by third parties are better than a surviving partner left with the uninvolved heirs of the decedent partner. Remembering the ordinal nature of these values, retaining that hierarchy while changing the values within specific bands is one way the parties can separately value their differing perspectives. This too is a simplification. One may allow values to move low-to-high as one progresses down the right side of the tree. Alternatively, values may be higher in the middle. There is no right answer necessarily, only a requirement that whatever method of valuing is employed be rational. Section III below will illustrate a broader range of choices while at the same time begging a series of questions as to why each choice is made.

Returning to the "Engaged," "Passive," and "Aggressive" approaches described in Section II.D above and their respective sets of payoff values and probabilities, the change in age, health, and circumstances of the two founding partners will create a natural division of their separate, personal goals and objectives. So, while at inception both partners have been "Engaged," the ravages of time might well have left one of them "Passive." With that assumption, it is evident that the passing of one partner's interest to heirs would widen the gap between partners even more, especially if one partner morphs into a group of heirs, each in varying conditions of age, health, and circumstances. All of this discussion is intended to justify the value of early negotiation among the founding partners, and recognize that "approaches," be there three or thirty-three of them, exist and come with their own particular sets of payoffs and probabilities to be inserted into the blanks on the decision tree.

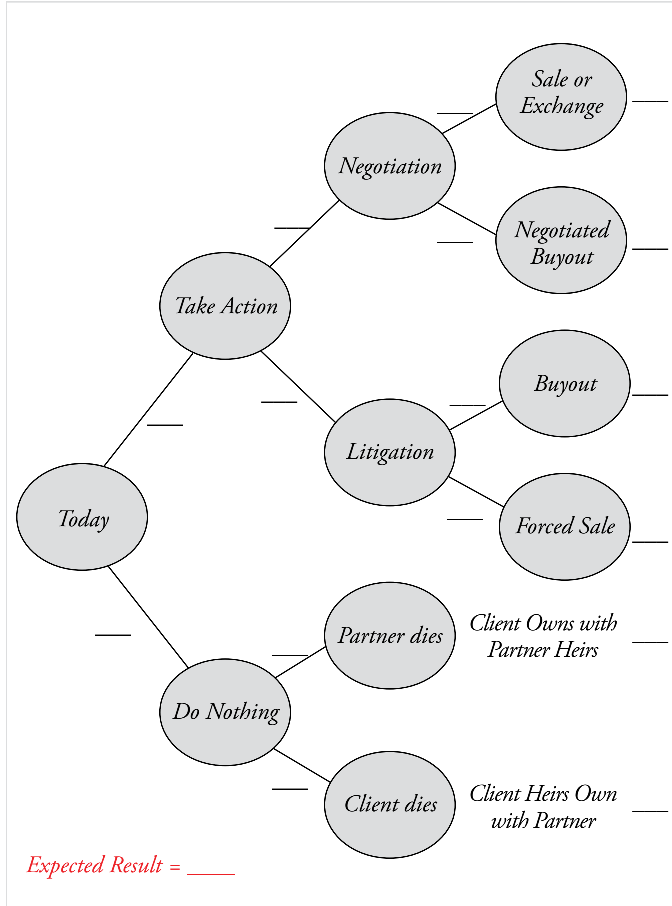

Figure 9

Blank decision tree template

III. AUTOMATING THE DECISION TREE CALCULATION

This piece now discusses the final and most functional feature of the decision tree calculator. The "Custom" tab in Figure 10 below uses the default starting values drawn from the "Engaged" option in Figure 8 in Section II.D above under the "Fixed/Random/Approach" tab (not selected in the static version of the automation tool shown in Figure 10). However, under the "Custom" tab shown in Figure 10 below, the user may set a broad range of values and probabilities with the expected result automatically calculated.

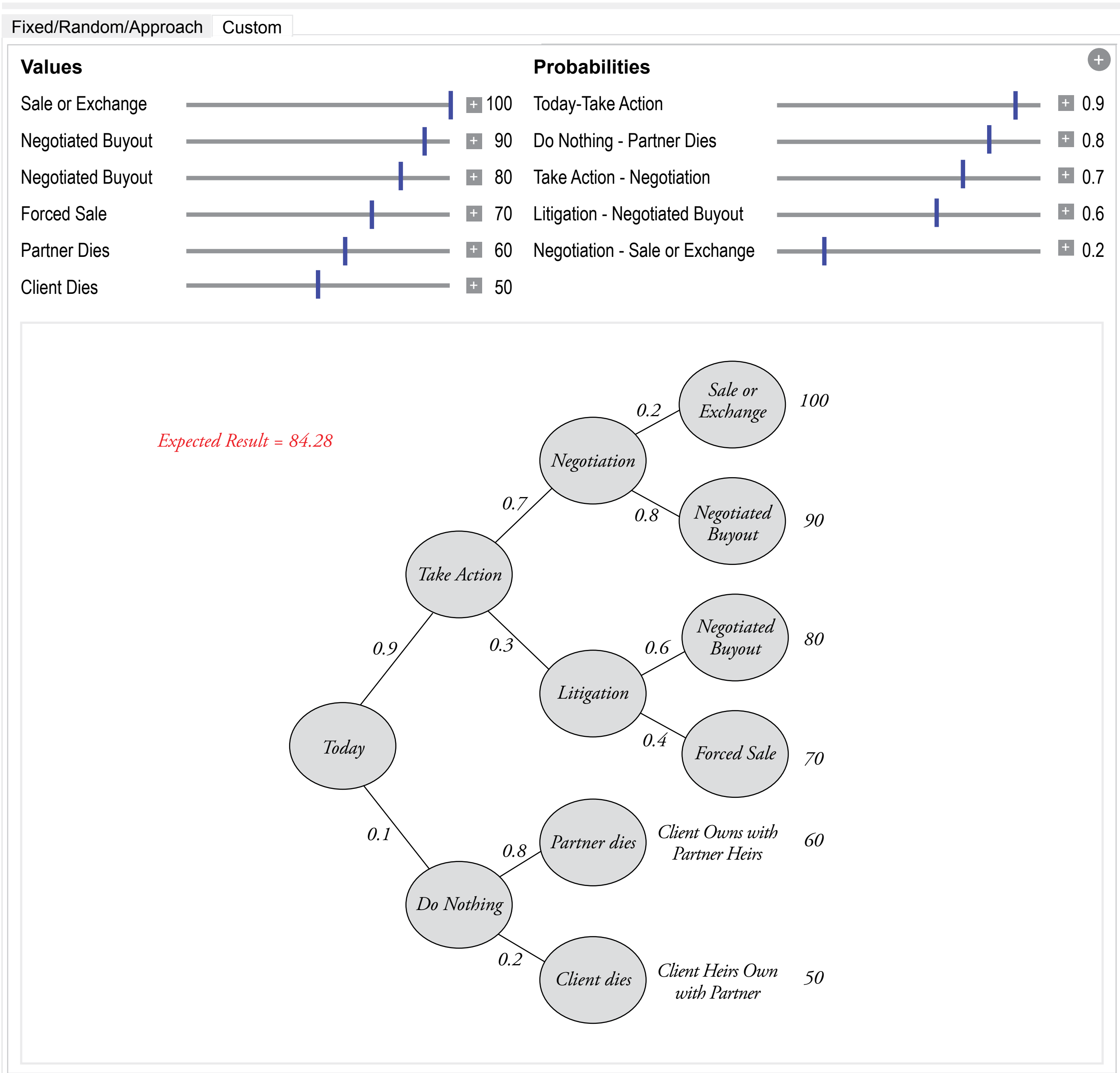

Figure 10

A robust decision tree calculator

Assuming the mediator is skilled in guiding the parties, only two rules must be followed. They are:

1. Each payoff value must be between zero and one hundred in increments of ten, and must descend from top to bottom along the right hand side;[21] and

2. Probabilities must be between zero and one in increments of 0.05 and each pair extending out from each node must add up to one.[22]

It is a near certainty that the parties will not reach the same number for the expected result if one assumes one's own death is less preferred than another's death. The purpose, however, is not to have the parties reach the same expected result; rather, it is to have them evaluate increments of the decision making process in the context of an overall outcome. This sort of reductionism has a long and distinguished history in human thought and analysis.[23]

With respect to the expected result, it is a fair question to ask: For whom is the expected result expected? Behavioral scientists can sort out when partnerships with joint and common goals and objectives devolve into sets of differing individual goals and objectives. It is at this flashpoint that the parties begin to count their separate winnings and come into conflict. Thus, the decision tree should apply to each decision maker individually. If used properly as a way to educate the parties, deflect them from emotional uproar, and place them on a path to objective values, the exercise can bear fruit.

IV. CONCLUSION

The discussion in Section II above hints at a more global answer to the question posed in Section I above. The reason machines learn is that they quantify things, follow rules, and seek an optimal solution. Certainly the programmer influences and even sets the goal the machine aims to solve. Whether humans are willing or even able to set aside emotions and become coldly analytical when their health, fortune, and family are involved remains an open question.

Before the decision tree became a popular tool in the digital age, it worked well in operations management systems to build a robust industrialized society. Underlying this story is a cautionary tale of how the laws of men pale when compared to the laws of nature. The logic of probability and economics of natural self-interest are powerful constructs which bow to no mortal. Perhaps the best attorney appeals to universal laws at an early stage. While this article has used mediation as its template and partnership dispute as its example, the cautious, transactional draftsman may profitably use the decision tree to encourage a carefully stated exit strategy in formation documents. The decision tree is just a more elaborate form of the old Ben Franklin Balance Sheet.[24] Artfully used early in the planning process, common sense and sound mathematics can help clients avert heartache and disaster in later years.

Endnotes

1 IMOJIM, Inc. PO Box 1146, Alpine CA 91903 rjb21@cox.net. The author wishes to thank Stevens Carey, Esq., Gary Laturno, Esq., James Pokorny, Esq., Jason Ebert, Esq., and Roy O. Williams for reading early drafts of this work and offering helpful comments. All errors remain solely those of the author.

2 This problem is most often found in simple co-tenancies where no partnership agreement exists, and in matters governed by Uniform General Partnership Law.

3 While the first use may be from an earlier date, it appears the idea first appeared in the literature in the late 1950s. cf. Rex V. Brown, *Do Managers Find Decision Theory Useful?*, Harv. Bus. Rev., May–Jun. 1970, at 78–89; Sidney W. Hess, *Swinging on the Branch of a Tree: Project Selection Applications*, Interfaces, Nov.–Dec. 1993, 5–12.

4 Airline pricing algorithms are taught by many universities. MIT offers a course which is considered part of its artificial intelligence program. *See* Notes for MIT Course 6.034 "Computational Complexity of Air Travel Planning" (Fall 2003), http://www.ai.mit.edu/courses/6.034f/psets/ps1/airtravel.pdf. The actual routine is random forests, for which decision trees are the progenitor. On September 7, 2016, a Google search on the terms "digital imaging random forests" yielded 293,000 hits.

5 In a partnership agreement, in California, partition rights are waivable. This right does not prevent conflict from ancillary derivative actions designed to destabilize the venture and promote dissolution.

6 Funds for the performance of this agreement may be provided by life insurance policies.

7 Use of decision trees is not new in the law. cf. John Zeleznikow & Dan Hunter, *Reasoning Paradigms in Legal Decision Support Systems*, 9 Artificial Intelligence Rev., 1995, at 361–85; Dan Hunter, *Near Knowledge: Inductive Learning Systems in Law*, 5 Va. J. L. and Tech., no. 9, 2000, at 361; Brian Daley, *Decision-Tree Analysis: An Effective Method to Manage Litigation in a Business Setting*, Passport for Business, Fall 2008; John DeGroote, *The Decision Tree, Step by Step: How Much Is Your Million Dollar Case Worth?*, Settlement Perspectives, November 9, 2011; Daniel M. Klein, of Klein Dispute Resolution, offers an online decision tree generator. *See* Daniel M. Klein, *Create a Decision Tree*, Klein Dispute Resolution, http://decisiontree.kleinmediation.com/tree/generator (last visited May 16, 2017).

8 In decision science the actions are referred to as "nodes" and the connecting lines are "vertices."

9 The reader familiar with the economic notion of "utility" will recognize this time-worn, if at times controversial, reasoning.

10 Let us not quibble at this point over the semantic difference between that term and "expected value" lest we become embroiled in an involved side trip through probability theory.

11 For simplicity, ignore a simultaneous death in a common disaster.

12 The application of mortality tables to a single life is frowned upon by actuaries for good reason. The validity of such tables arises from the use of large numbers.

13 The reader with substantial experience in estate planning or homicide could pose strong objections. Anecdotal evidence may always be found to contradict any general rule. Rather than become impaled on this, maintain patience just a bit longer to allow the process to be fully developed.

14 The electronic version of this object may be downloaded at: MATHESTATE, http://www.mathestate.com/DecisionTree.cdf (last visited May 16, 2017). It may be operated by using the free Wolfram CDF Player which may be downloaded at: WOLFRAM, http://education.wolfram.com/cdf-player-download.html (last visited May 16, 2017).

15 The program that operates the dynamic version of the static graphic in Figure 8 picks random numbers in the appropriate range to effectuate randomly different results.

16 Attributed variously to Enrico Fermi or Stanislaw Ulam, physicists on the Manhattan Project, the term "Monte Carlo" has long been used as a meme for various computational methods that simulate random outcomes. With the advent of fast, inexpensive desktop computation, it is now in wide use as a risk management tool. In our case, the approach is particularly useful since gathering large amounts of data is difficult or impossible.

17 Recall the great philosopher Woody Allen who said, "I am not afraid of death, I just don't want to be there when it happens."Woody Allen, *Death, in* WITHOUT FEATHERS, Random House, 1975.

18 If this sounds like profiling, it is because it is. Profiling is just a special case of conditional probability.

19 In fact, they are influenced by the author's nearly half century of experience dealing with such matters involving real people and real assets.

20 In the interests of space, the three different approaches are not illustrated. Figure 8 shows the drop down menu before a choice is made. The Expected result for "Engaged" is 84.28; for "Aggressive" is 76.60; and for "Passive" is 58.60.

21 Relaxing "must" to "may" is allowed, with the caution described earlier about making the order rational.

22 The fineness of the gradation may be set to allow for more variation in either payoff or probability input.

23 The Stanford Encyclopedia of Philosophy devotes considerable space to this time-worn method. *See Scientific Reduction*, STAN. ENCYCLOPEDIA OF PHILOSOPHY (Apr. 8, 2014), https://plato.stanford.edu/entries/scientific-reduction/#ModSciRedSci.

24 BENJAMIN FRANKLIN, *To Joseph Priestly*, *in* THE PAPERS OF BENJAMIN FRANKLIN, 299–300 (William Bradford Willcox ed., 1975).