MODELING FINANCIAL MARKET RETURNS WITH A LOGNORMALLY SCALED STABLE DISTRIBUTION

RIMMER, ROBERT H. (speaker), *United States*
BROWN, ROGER J., *United States*

Financial Market Returns; Stable; Lognormal; Extreme Value;

A stable mixture distribution is presented as a model for intermediate range financial logarithmic returns. The model is developed from the observation of high frequency one minute market returns, which can be well modeled by random noise generated by a stable distribution multiplied by a non-random market parameter, which is a measure of market volatility. The stable distribution has an α parameter of approximately 1.8, for the actively traded ETF, SPY. The daily time series of the scale factor shows strong serial dependence. Nevertheless the daily scale factor over periods of months to years is well fit by a lognormal distribution. Thus intermediate term market simulation and risk modeling can be accomplished with the product of a lognormal random variable and a standardized stable random variable. Although there is not a closed formula for the stable distribution, the mixture distribution and density functions can be approximated by numerical integration. Where ϕ is the stable characteristic function, and λ is a lognormal density, the mixture characteristic function can be given by mcf.

$$\phi(t, \alpha, \beta) = e^{-|t|^{\alpha}\left(1 - i\,\beta\,\text{sgn}(t)\tan\left(\frac{\pi\alpha}{2}\right)\right)}$$

$$\lambda(x, \mu, \sigma) = \frac{e^{-\frac{(\log(x)-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi}\; x\,\sigma}$$

$\text{mcf}(t, \alpha, \beta, \gamma, \sigma, \delta) = e^{i\,\delta\,t}\int_0^{\infty}\lambda(s, \log(\gamma), \sigma)\,\phi(s\,t, \alpha, \beta)\,ds$, where α is the shape parameter of the stable distribution, β is the stable skewness parameter, γ is the median of the scale factor distribution, δ is a location parameter, and σ is the shape parameter of the lognormal distribution fitting the varying scale factor. Numerically it is difficult to fit these parameters to data, but with the large sample sizes provided by intra-day minute data, α can be approximated using the generalized extreme value distribution, and maxima of partitioned data. α can also be approximated by sequentially fitting each day's data; this value is surprisingly consistent, or by rescaling each day's data by the stable γ for the day and performing a stable fit to the rescaled data. The parameters for lower frequency daily returns can be approximated by taking advantage of the serial dependence, estimating the scale factor for partitioned data and rescaling the partitions.

The presentation shows evidence for the model with one minute returns of the SPY ETF collected since July 2007. This time frame includes a rather remarkable variation in market volatility, yet the model seems to remain valid. Calculations of the functions are demonstrated with *Mathematica*, and John Nolan's program, STABLE. A web resource of programs in *Mathematica* will be made available.

The model is attractive since it can account for all the stylized facts about financial returns and be explained as arising from the behavior of a continuous double auction market model that has limit order book return distributions with heavy power-tails, which over very short times measured in seconds yield independent returns obeying the generalized central limit theorem. The varying scale factor or volatility accounts for the serial dependence seen in the absolute value of market returns. The density of the mixture distribution has a higher peak than a stable distribution with the same parameters, α, β, γ, but on the tails it asymptotically approaches a stable distribution. Thus it is different from a truncated stable distribution. Sums of independent random variables from this distribution will converge to a stable distribution, but such behavior may not be observed in financial data because the scaling variables are not independent.

References

[1] Nolan, J. (2009) Stable Distributions - Models for Heavy Tailed Data, Birkhauser. Note: in progress, Chapter 1, available at: http://academic2.american.edu/~jpnolan/stable/stable.html

[2] Embrechts, P., Klüppelberg, C., Mikosch, T. (2008) Modelling Extremal Events for Insurance and Finance, Springer.

[3] Feller, W. (1971) An Introduction to Probability Theory, Vol. II, Wiley.

[4] Nolan, J., STABLE, a program for calculating stable distributions, with interfaces to matlab, Mathematica, S-Plus, and R. http://www.robustanalysis.com