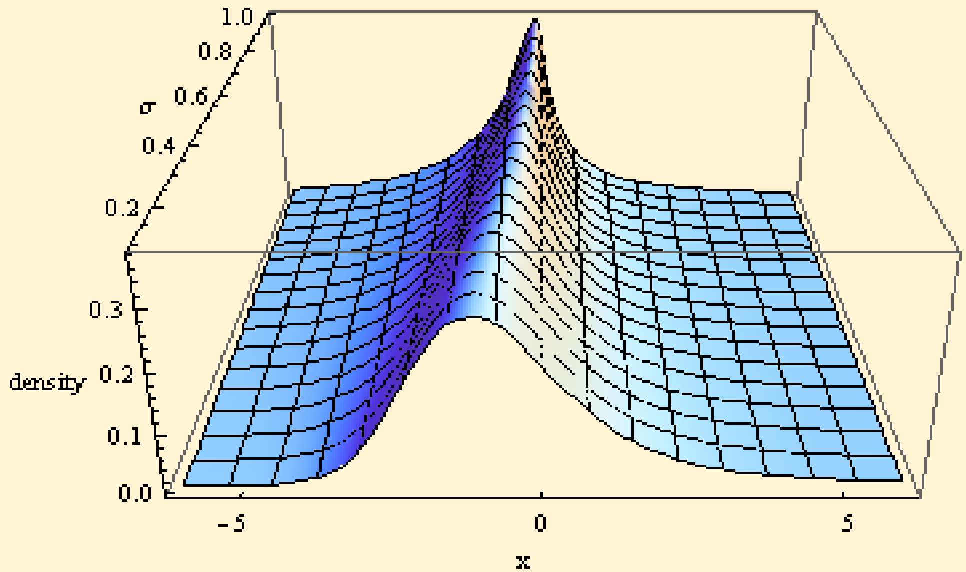
Modeling Financial Market Returns with a Lognormally Scaled Stable Distribution

Robert H. Rimmer
Roger J. Brown



STABLE software by John Nolan
www.robustanalysis.com

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Market Model: Stable Mixture Distribution with varying scale factor

Product of Two Variables

Stable random variable

Scaling variable that is not independent
Density close to lognormal

Stable Characteristic Function

$$\phi(t) = \exp\left(i\, t\, \delta - \gamma^{\alpha}\, |t|^{\alpha}\left(1 - i\, \beta\, \mathrm{sgn}(t) \tan\left(\frac{\pi\, \alpha}{2}\right)\right)\right); \qquad \alpha \neq 1$$

$$\phi(t) = \exp\left(i\, t\, \delta - \gamma\, |t|\left(1 + \frac{2\, i\, \beta\, \mathrm{sgn}(t) \log(|t|)}{\pi}\right)\right); \qquad \alpha = 1$$

α is the shape parameter — $\alpha \in (0, 2]$
tail exponent ($\alpha < 2$)
β is the skewness parameter — $\beta \in [-1, 1]$
γ is the scale parameter — $\gamma \in (0, \infty)$
δ is the location parameter — $\delta \in$ Reals

Limiting distribution of sums of i.i.d. RVs

Distribution of sums of stable RVs is stable with same α and β
scaling is by n^(1 / α)

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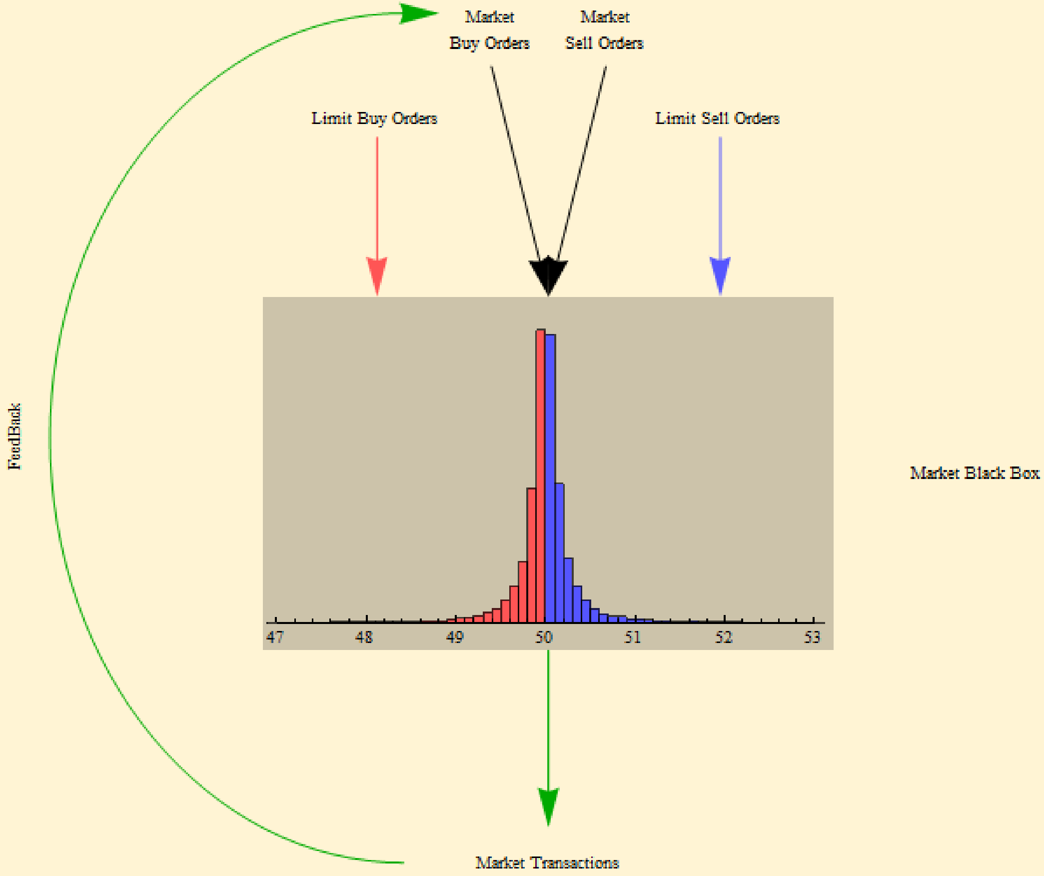
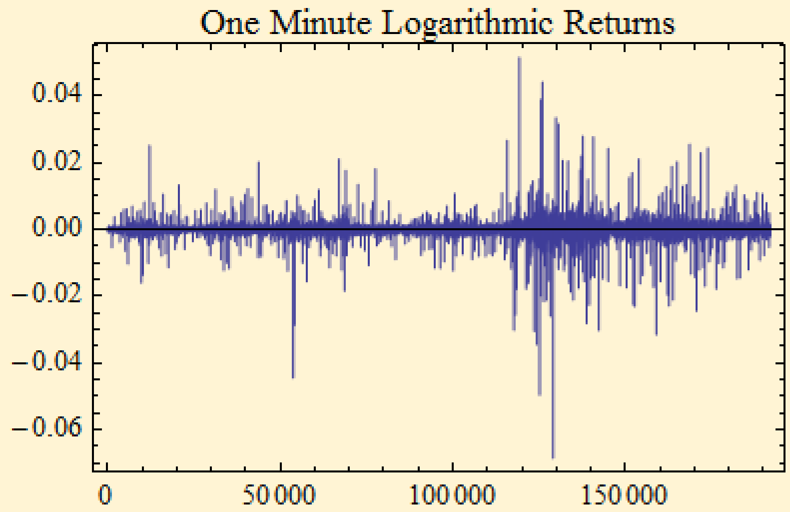
Continuous Double Auction



Under controlled conditions CDA can output stable RVs

SPY ETF Daily Closing Prices
Logarithmic Returns



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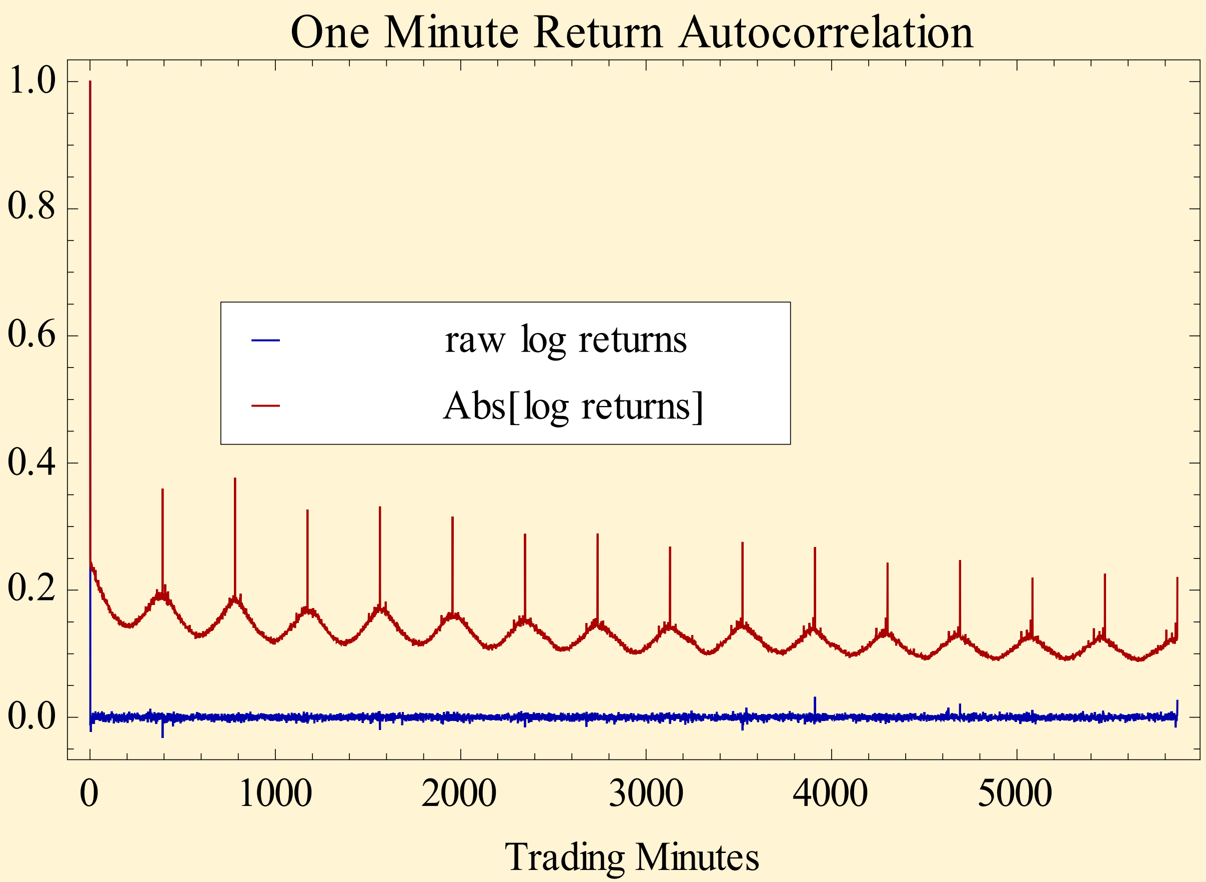
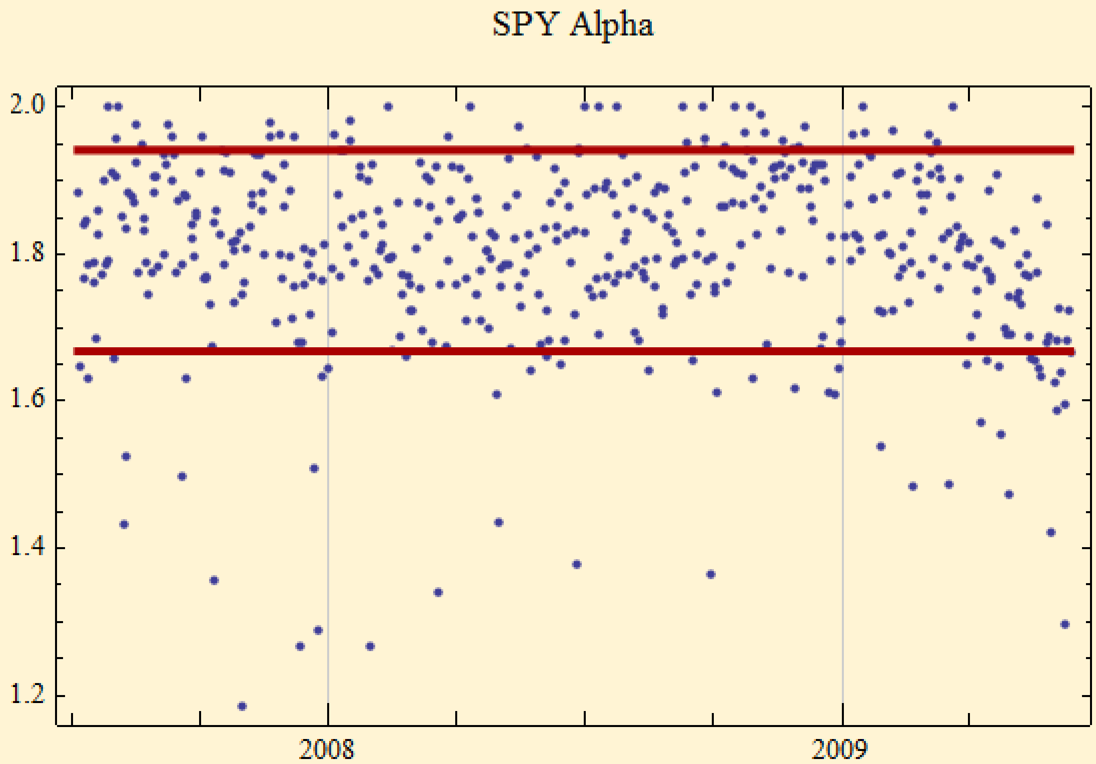
Autocorrelation Absolute Value Returns

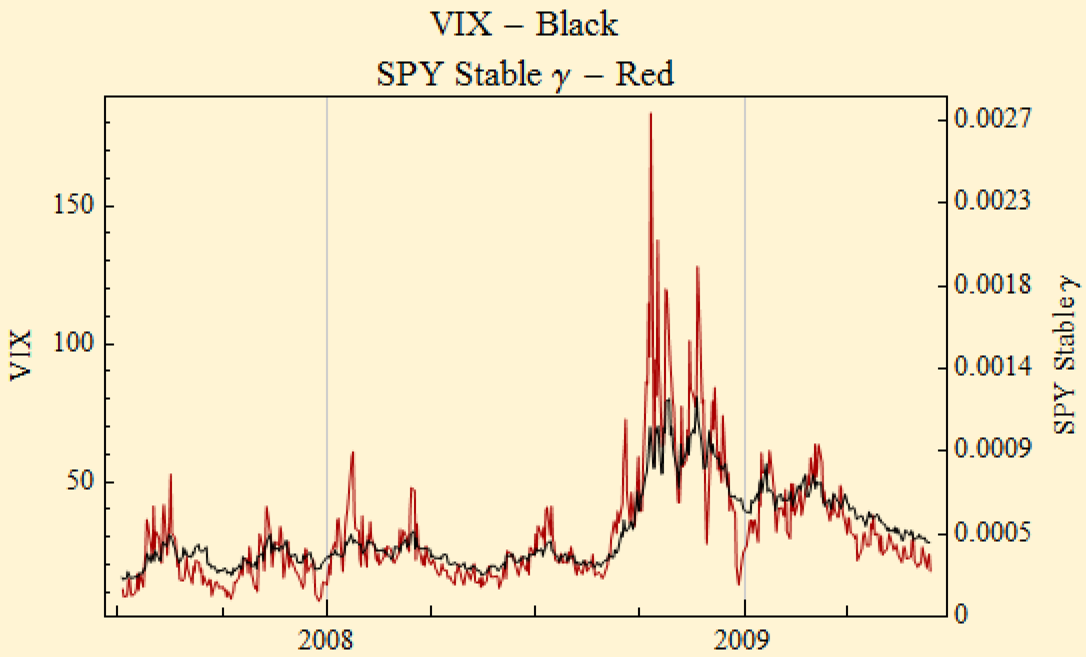


Daily Stable Parameters

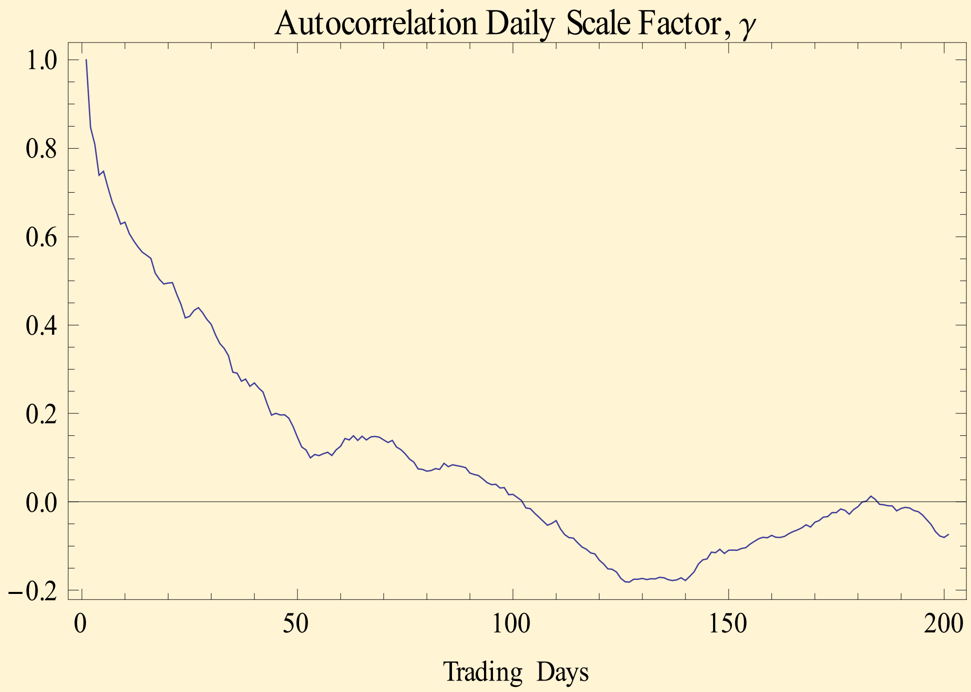
α	β	γ	δ
1.8052	0.0370129	0.000483713	5.80342×10^{-8}



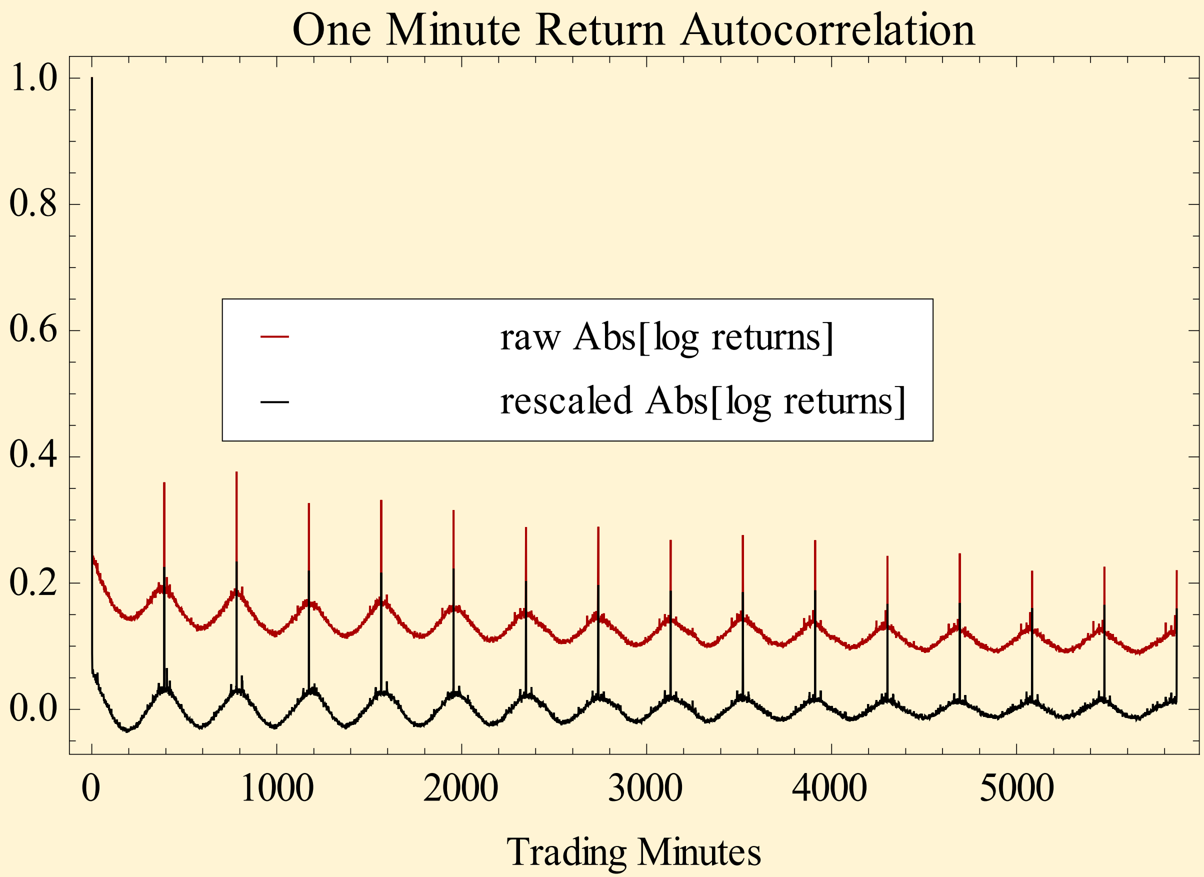
Stable Gamma
a measure of volatility



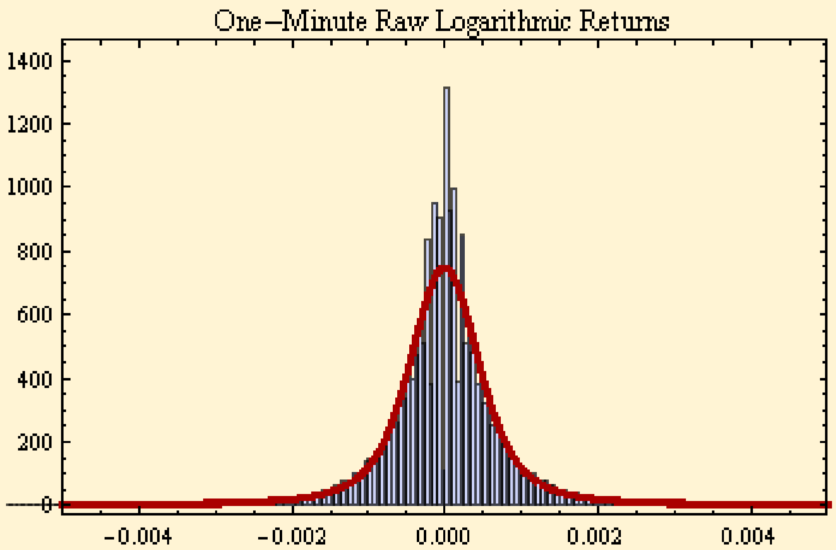
Volatility Autocorrelation



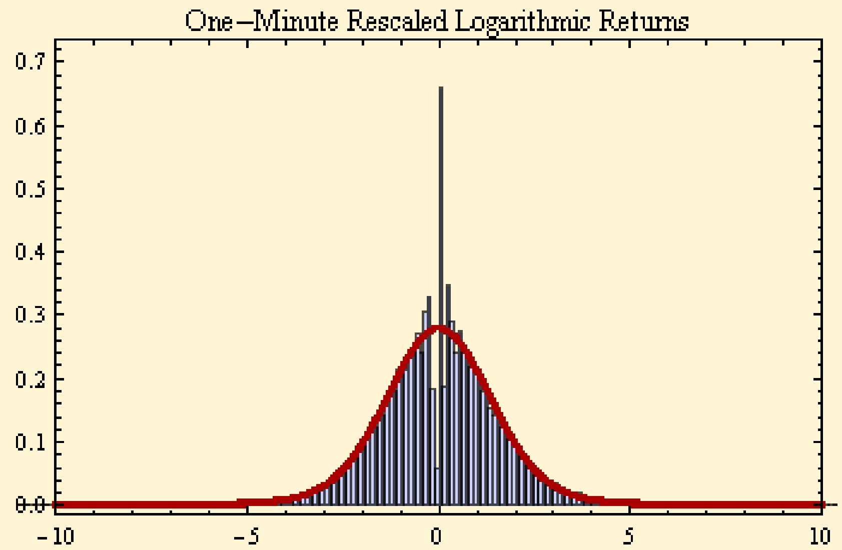
Autocorrelation Rescaled Returns



Stable Fits



α	β	γ	δ
1.41694	1.26061×10^{-8}	0.000387194	-3.73386×10^{-6}



α	β	γ	δ
1.79027	-3.25178×10^{-9}	1.014	-0.00312431

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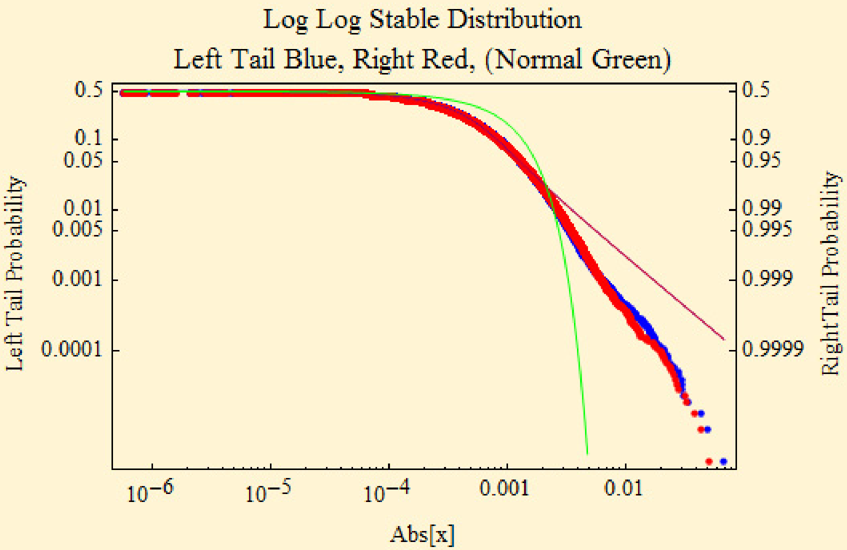
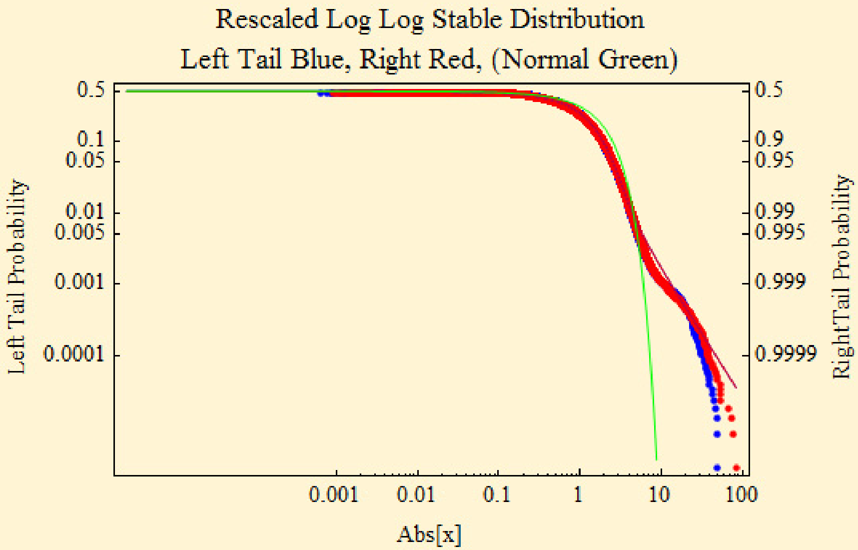
Stable Tail Fits





α	β	γ	δ
1.41694	1.26061×10^{-8}	0.000387194	-3.73386×10^{-6}

α	β	γ	δ
1.79027	-3.25178×10^{-9}	1.014	-0.00312431

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Lognormal Density

$$\lambda(x, \mu, \sigma) = \frac{e^{-\frac{(\log(x)-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi}\, x\sigma}$$

Stable Characteristic Function

$$\phi(t, \alpha, \beta) = e^{-|t|^\alpha \left(1 - i\beta\, \mathrm{sgn}(t) \tan\left(\frac{\pi\alpha}{2}\right)\right)}$$

Lognormal Stable Characteristic Function

$$\mathrm{lnscf}(t, \alpha, \beta, \gamma, \sigma, \delta) = e^{i\delta t} \int_0^\infty \lambda(s, \log(\gamma), \sigma)\, \phi(s\,t, \alpha, \beta)\, ds$$

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Lognormal Stable Distribution

$$\text{lnscdf}(x,\ \alpha,\ \beta,\ \gamma,\ \sigma,\ \delta) = \int_0^{\infty} \text{scdf}(x,\ \alpha,\ \beta,\ s,\ \delta)\,\lambda(s,\ \log(\gamma),\ \sigma)\,ds$$

Lognormal Stable Density

$$\text{lnspdf}(x,\ \alpha,\ \beta,\ \gamma,\ \sigma,\ \delta) = \int_0^{\infty} \text{spdf}(x,\ \alpha,\ \beta,\ s,\ \delta)\,\lambda(s,\ \log(\gamma),\ \sigma)\,ds$$

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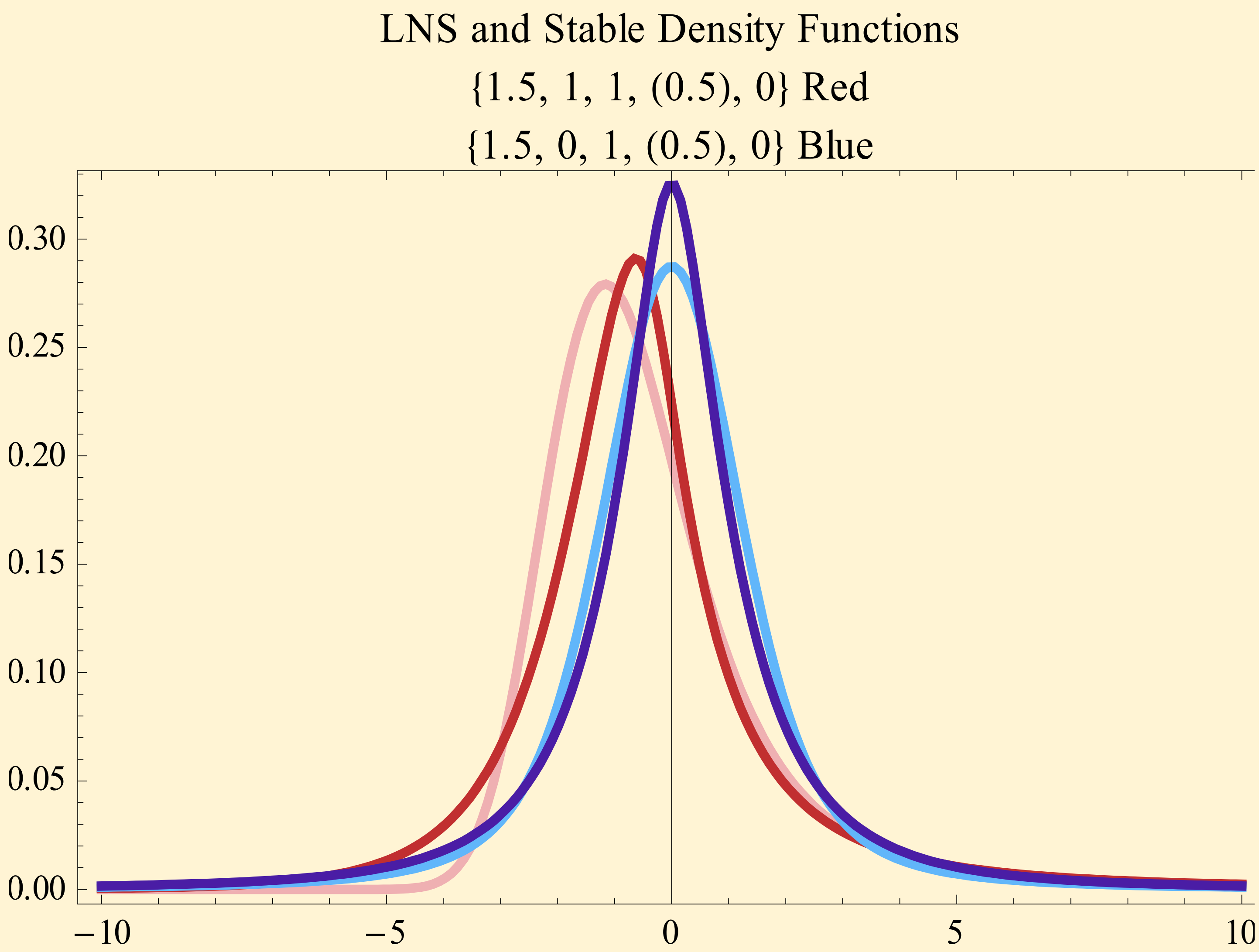
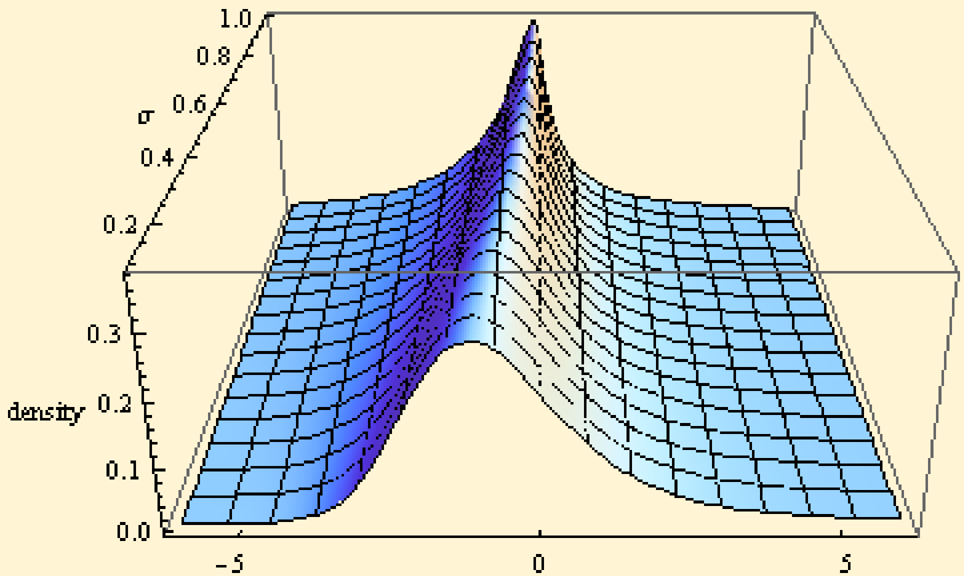
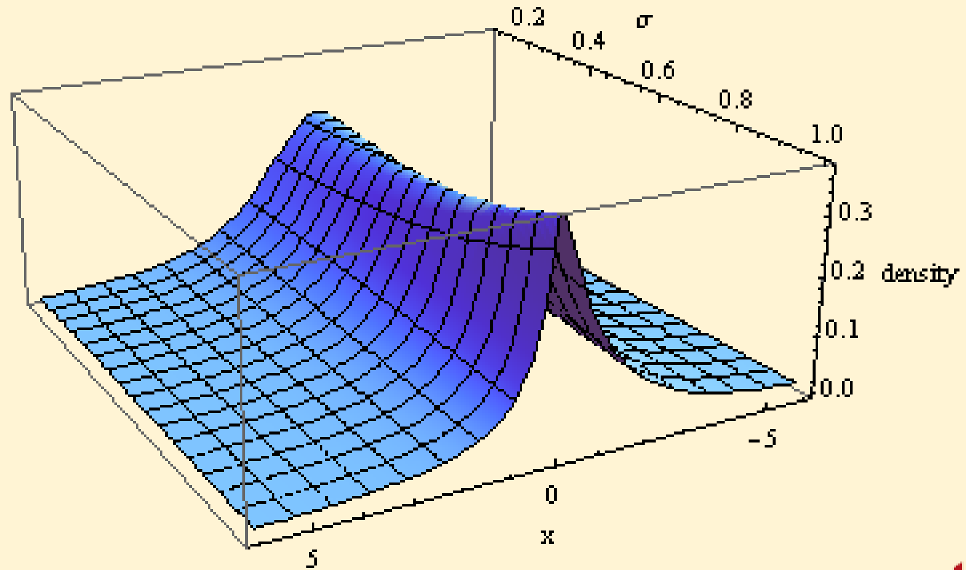
Lognormal Stable Density



Lognormally Scaled Stable Distribution





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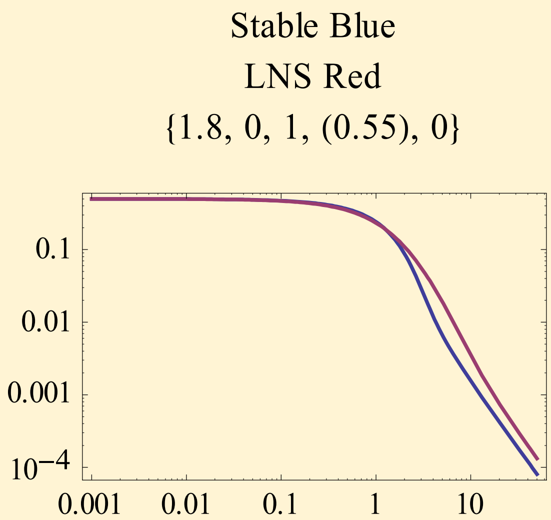
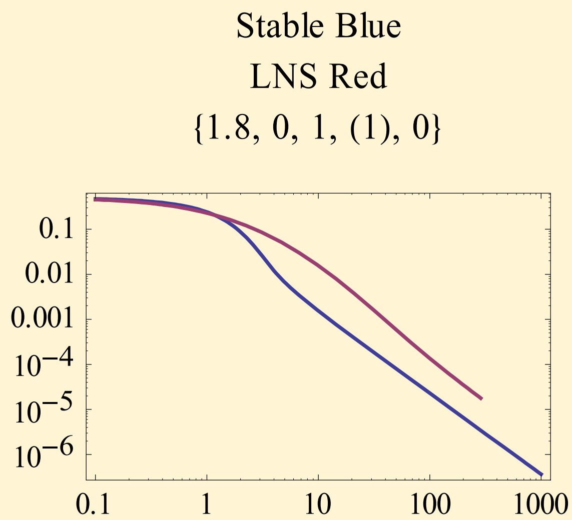
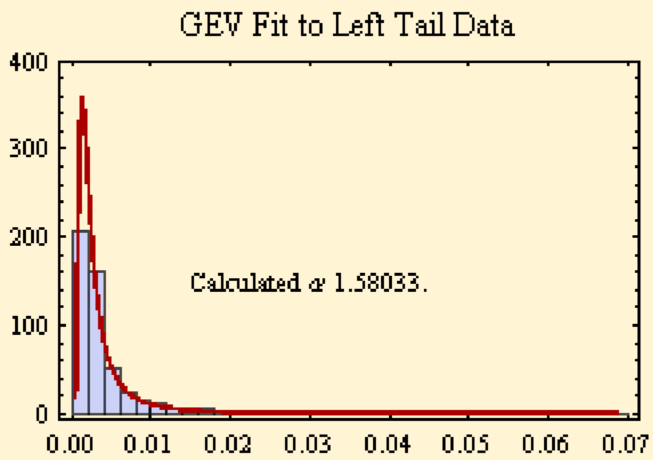
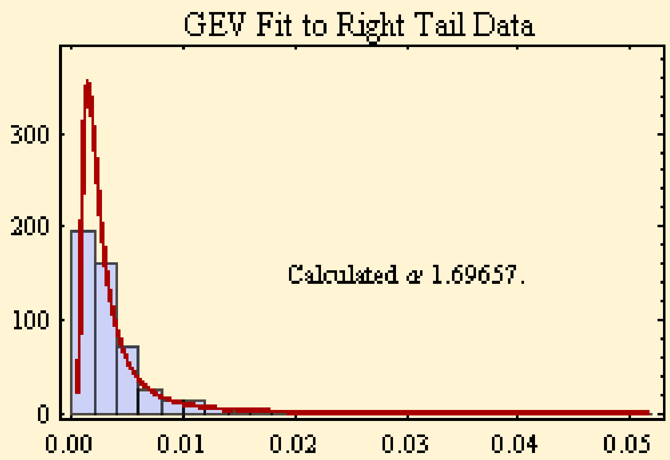
LNS Tail Behavior (CDF)





Estimating Tail Exponent with GEV





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Market Model

Product of stable random and a non-random scaling variable

This is a stable mixture distribution with a varying gamma parameter.

For financial data the scaling appears to be constrained; the tail exponent of the distribution can be estimated.

The model solves the fitting problems associated with the assumption of a stationary stable model.

It is possible to rescale daily data, taking advantage of the serial dependence in gamma.

If you can guess the future behavior of volatility, you can make some reasonable estimates of future event probability using stable distributions.

Lognormally Scaled Stable Distribution

Product of a stable random variable and a lognormal random variable.

It is computable.

The maximum domain of attraction of the distribution is determined by stable alpha.

Can be used for simulation.

Serial dependent structure can be added to the lognormal RV to better understand financial data.

For a more detailed technical paper on this presentation visit **www.mathestate.com**

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