Journal of Property Investment &
Finance
Vol. 35 No. 3, 2017
pp. 341-351

Entropy: What Kind of Bet is Real Estate - Really?

Abstract

Accepted orthodoxy has it that real estate markets are inefficient. Despite decades of failed attempts to pound the square peg of real estate through the round hole of finance, the temptation remains to genuflect to the dominant paradigm. With the digitization of the industry, the advent of big data, and a host of data sources, real estate has yet to find its fit amidst the avalanche of ones and zeroes.

This research effort inquires into whether entropy explains real estate market inefficiency. In thermodynamics, entropy is always rising. In a closed physical system equilibrium can be obtained. Clearly economics, human behavior and the exchange of information about assets do not constitute a closed system. If real estate information entropy is always rising, why should we ever expect real estate markets to be efficient?

Kelly (1956) postulated an investment strategy based, in part, on information science. His method dominated other, popular, finance methods in the context of securities. Until now this technology has not been applied to real estate investment. This paper explains why.

Keywords: Entropy; information theory; real estate and uncertainty; Kelly criterion

Journal of Property Investment & Finance
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Entropy: What Kind of Bet is Real Estate - Really?

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It is never worth a first class man's time to express a majority opinion. By definition, there are plenty of others to do that. -- G. H. Hardy[2]

I. Introduction

What can information theory tell us about real estate?

Entropy has been understood by physicists for 100 years. Over more than half of that time, a generalization of the concept to information theory (Shannon, 1948) has been rewarded with expansions in communication and market theory. Shannon once mused that "Extensions [about his work in communications] to other fields were suspect."[3] At the risk of just that, this paper attempts to extend entropy and information science to real estate in the context of games of chance while focused on how real estate information propagates through the system we know as the real estate market.

In recent decades virtually all industries have, to one degree or another, digitized. For this we have advances in information theory to thank, which suggests that the universe can be described as a series of ones and zeroes. Alternate forms are yes/no, black/white. Even fifty shades of gray may be seen as an ordered list of 49 questions "Darker? (Y/N)." From recent history we tend toward the conclusion that our world is binary.

Real estate suffers from a kind of epistemological schizophrenia in that it wants to join the Information Age, but on the condition that it discloses only the zeroes and keeps the ones to its own clan. The 21st Century wants information to flow freely. Real estate wants to perpetuate its long history of playing "Hide the ball." The conflict is similar to what has affected the music, publishing and entertainment industries, as well as the education establishment. Even national security has been

1 Colvin Institute of Real Estate Development, University of Maryland at College Park, College Park, Maryland, USA. The author would like to thank Marlyn L. Hicks, Kevin Dowd and William Wu for very helpful comments. All errors remain the author's solely.

2 Gaither, Carl C.; Cavazos-Gaither, Alma E. (2012). *Gaither's Dictionary of Scientific Quotations*. Springer. p. 1645.

3 Private communication with Shannon discussed in Tribus (1983) as quoted in Ritchie (1986). Harry Markowitz would be justified in expressing the same sentiment after seeing how Modern Portfolio Theory was "extended" to include real estate.

forced to grapple with these realities. We have learned, or should have learned, that once something of value is digitized and transmitted through satellites, over networks and stored in a virtual cloud, it is very hard to keep it contained.

II. Literature review

Maxwell (1872), Boltzman (1896) and Gibbs (1902), bringing statistical mechanics to thermodynamics, first introduced entropy as the loss that occurs when energy is converted to work. In a closed system entropy only rises until equilibrium is reached. Since the beginning of the 20th century entropy has also, at times, been used to describe or claimed to be capable of portraying uncertainty or disorder.

Shannon (1948), in the context of electrical engineering and the problem of compressing and sending data over power lines, generalized the idea to information, showing that predictability in communications can be quantified. Shannon entropy measures the degree of uncertainty in a random variable. Many have since objected that Shannon's work was in the very narrow area of pulses sent over wires and that expanding his work into other fields is misguided. Nonetheless, expanding is precisely what has been done.

Eigen and Winkler (1965) characterized Nature as governing chance, not the other way around as is often claimed. They make the point that entropy is a dimensionless variable of state represented by the logarithm of the number of possible combinations of symbols. They caution that its use should be restricted to conditions "...where average values convey meaningful information."[4]

Kelly (1956) noticed that the product of random variables leads to the geometric mean and argued for a finance application that produced a portfolio solution which took issue with Markowitz (1952). Breiman (1961) proved that Kelly's method offered the optimal time horizon. Samuelson (1969 and 1979) took exception based on a broader class of utility functions.

Bell and Cover (1980) showed that Kelly offered competitive optimality. Barron and Cover (1988), elaborating on mutual information, established a bound on growth offered by the Kelly Criterion.

Bais and Farmer (2007) offer a comprehensive approach that connects physics and information, adding important links to fractal geometry and quantum measures.

Three particular works should be required reading for anyone enamored of Shannon entropy to the point of adopting it for explanations outside the field for which it was created. The first is Sokal and Bricmont (1999)[5] and covers the most general of the problem of mapping concepts from one field to another. The second, Losee (1998), and third, Ritchie (1986) are more specific with the latter bordering on a blistering indictment.

[4] Page 149 of the 1981 translation.

[5] Anyone not aware of this brilliant hoax is missing not only a great lesson but a lot of fun. The book offers insight as well as a cautionary tale for those in the intellectual borrowing business.

Nearing a way station on this road we begin to get a picture of how the evolution of thinking affects reality and vice versa. It is no doubt true that Claude Shannon meant his theory to apply only to electrical pulses sent over copper wires. But he may not have imagined the pervasively wireless world of the 21 Century. The present stop on that evolution has us recognizing that *the nature of meaning is whether the ones and zeroes are all present and in the right order*. The word "whether" in the foregoing sentence presents a probability question. What we contemplate today is a mathematical foundation for Marshall McLuhan's famous line "The medium is the message".

III. Shannon information

The broadest idea of information and uncertainty is set theory, picking *r* particular objects from a pool of *n* such objects. This binomial problem combined with Bayesian methodology represents the launch point for the mechanical communication Shannon described.

The essence of Shannon information theory is data compression. Shannon showed that there is an algorithm which minimizes the number of characters required to unambiguously instruct a computer. The model is

Sender – channel – receiver

where the signal sent by the sender through a noisy channel is binary, hence in the form of bits. Under carefully stated but not terribly restrictive assumptions one may code a message in such a way as to make the probability of error at the receiving end as small as desired. A central result, information entropy (H), is:

$$H(p) = -\sum p_i Log_2(p_i) \qquad (1)$$

where p_i = probability of an individual symbol. This, in a Shannon world, is a measure of *whether* on average the information received is the same as the information sent.

Two important observations are useful at this point. One is that Eq. (1) is discrete[6] and the other is that entropy is a function of probability (p). The stretch that would raise Shannon's eyebrows when we extend his work to real estate is broadening the probability function from *if* the information appears to *whether information is released*. Unlike thermodynamics where entropy never decreases, information entropy may decrease as uncertainty is resolved.[7]

Surprisal is a measure that describes how astonished we are when we learn something. Naturally, surprisal grows larger as the probability decreases. The expression for surprisal, Eq. (2), is part of Eq. (1):[8]

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[6] Results are not exactly the same in the continuous case

[7] The author is indebted to Martin Zwick for pointing this out.

[8] The minus sign is to make the result a positive number since the Log of a probability is negative as it is the Log of a number between 0 and 1. Equivalently $u(p) = Log_2(1/p_i)$.

$$u(p) = -Log_2(p_i) \tag{2}$$

Examples of entropy using Eq. (1) are useful. Assume a message in English involves the use of no more than thirty characters, 26 letters, a space and 3 punctuation marks. If the message is merely those characters: “abcde fghij;klmno.pqrst:uvwxyz”, Table 1 shows how Eq. (1) operates. There are 30 unique characters and entropy is as high as possible because the appearance of each character in the message is equally probable. Thus, the probability mass function (PMF) has 30 identical fractions, each 1/30.

number of characters	30
character tally	{{a, 1}, {b, 1}, {c, 1}, {d, 1}, {e, 1}, { , 1}, {f, 1}, {g, 1}, {h, 1}, {i, 1}, {j, 1}, {;, 1}, {k, 1}, {l, 1}, {m, 1}, {n, 1}, {o, 1}, {., 1}, {p, 1}, {q, 1}, {r, 1}, {s, 1}, {t, 1}, {:, 1}, {u, 1}, {v, 1}, {w, 1}, {x, 1}, {y, 1}, {z, 1}}
PMF	$\left\{\frac{1}{30}, \frac{1}{30}\right\}$
PMF sum	1
unique characters	30
entropy	4.90689

Table 1 – Entropy of 30 equiprobable unique characters

When characters appear more than once in the same message, entropy falls. Assume the message is “this sentence is 30 characters”. Table 2 shows how entropy is affected when some characters appear as many as four times.

number of characters	30
character tally	{{t, 3}, {h, 2}, {i, 2}, {s, 4}, { , 4}, {e, 4}, {n, 2}, {c, 3}, {3, 1}, {0, 1}, {a, 2}, {r, 2}}
PMF	$\left\{\frac{1}{10}, \frac{1}{15}, \frac{1}{15}, \frac{2}{15}, \frac{2}{15}, \frac{2}{15}, \frac{1}{15}, \frac{1}{10}, \frac{1}{30}, \frac{1}{30}, \frac{1}{15}, \frac{1}{15}\right\}$
PMF sum	1
unique characters	12
entropy	3.45656

Table 2 – Entropy of 30 non-unique characters

A lively debate, still unresolved, over the definition of communication includes whether any part of Shannon or his entropy involved “meaning” in the sense of what might be called “effective communication”. This debate involves confusion and a paradox. Shannon may rightly claim that a message received at the end of a channel which precisely matches that which was sent means that the communication was effective. In the sense of the message sent as illustrated in Table 1, this can be confirmed only if the goal of the message was to send 30 unique characters. If the message was intended to confirm the time we are meeting for dinner it is rather ineffective. Communication, it turns out, involves human beings. Therefore “meaning” enters the discussion.

We need not assume that meaning plays a role. Note that “this sentence is 03 characters” produces the same result as Table 2 despite the fact that, should it have any meaning at all, that meaning is false.

We can lower entropy in two ways. One is to merely use fewer characters. The sentence "this phrase is 28 characters" produces the result in Table 3.

number of characters	28
character tally	{{t, 2}, {h, 3}, {i, 2}, {s, 4}, { , 4}, {p, 1}, {r, 3}, {a, 3}, {e, 2}, {3, 1}, {0, 1}, {c, 2}}
PMF	$\{\frac{1}{14}, \frac{3}{28}, \frac{1}{14}, \frac{1}{7}, \frac{1}{7}, \frac{1}{28}, \frac{3}{28}, \frac{3}{28}, \frac{1}{14}, \frac{1}{28}, \frac{1}{28}, \frac{1}{14}\}$
PMF sum	1
unique characters	12
entropy	3.44076

Table 3 – Entropy of 28 characters; 12 unique

The other method is to use fewer *unique* characters. Hence, a sentence of the same length, "this pppppp is 30 cccccccccc", produces lower entropy in Table 4.

number of characters	28
character tally	{{t, 1}, {h, 1}, {i, 2}, {s, 2}, { , 4}, {p, 6}, {3, 1}, {0, 1}, {c, 10}}
PMF	$\{\frac{1}{28}, \frac{1}{28}, \frac{1}{14}, \frac{1}{14}, \frac{1}{7}, \frac{3}{14}, \frac{1}{28}, \frac{1}{28}, \frac{5}{14}\}$
PMF sum	1
unique characters	9
entropy	2.63846

Table 4 – Entropy of 28 characters; 9 unique

The important ideas of relative entropy and mutual information flow from entropy.[9]

Relative entropy, D(p||q), is a measure of inefficiency resulting from assuming the wrong distribution.[10]

$$D(p||q) = \sum_{x \in X} p(x) log \frac{p(x)}{q(x)} \qquad (3)$$

Where

p(x) = the true distribution
q(x) = the (incorrect) assumed distribution

The simplest example of this is assuming a game is fair,

q(x) where X∈{0,1}, q(0)=.5; q(1)=1-.5=.5

when in fact it is favorable,

p(x) where X∈{0,1}, p(0)=.6; p(1)=1-.6=.4

Thus relative entropy is (in bits)

$$D(p||q) = (1-0.6)\ Log_2 \frac{1-0.6}{1-0.5} + 0.6 Log_2 \frac{0.6}{0.5} = .0290494$$

9 Examples in this section are drawn from Cover and Thomas (2006)

10 Also known as the Kullback-Leibler distance, this inefficiency in the Shannon sense has to do with coding length. A broader indictment elsewhere would be the common and often incorrect assumption of normality.

Mutual information, I(X;Y), is the reduction in uncertainty in X as the result of the knowledge of Y and is the relative entropy between the joint distribution (X ,Y) and the product distribution p(x)p(y), the simplest representation of which is, Eq. (4), the entropy of X less the conditional entropy of X given Y:

$$I(X;Y) = H(X) - H(X|Y) \qquad (4)$$

In advantage gaming parlance the common term for Y is "side information". Of course the use of side information in the stock market can lead to jail time. Real estate does not bear that burden.[11] The exploitation of side information is not only common in private real estate markets it is expected of sophisticated players. Hence, we have yet another reason to separate academic real estate forever and always from academic finance.

It has been observed that modern real estate practice exists at the intersection of law and economics. On the ground in the real world that process reveals itself as due diligence. This effort purports to uncover side information the seller failed to disclose to the buyer. Information theory productively exploits the Bayesian idea of updating the information set to reach posterior probability. The alert reader should note rich analogs between these ideas and private real estate investment. [12]

IV. The Kelly Criterion

Information theory and finance meet at a concept known as the Kelly criterion or Kelly betting. In trading vernacular it is known as "edge over odds" meaning that it is the ratio of the probability of a win (expected return) to the payoff probabilities. It has been shown that one limits exposure (using historical returns) as the Kelly criterion offers protection against ruin.[13]

There are several interpretations of the Kelly criterion. The simplest of these is that portion of one's capital wagered on a particular bet:[14]

$$\text{Kelly criterion}_1 = k(p,q,w,l) = p - \frac{q}{w/l} \qquad (5)$$

where

p = number of wins divided by the total number of bets
q = 1-p
w = average win payoff
l = absolute value of average loss

A little time spent with Eq. 5 will convince the reader that positive amounts of capital are only devoted to those situations where there is either (a) favorable odds ($p > .5$),

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11 Unless that real estate is securitized which is just another reason to treat REITs and other securities nominally based on real estate as stock and not real estate.

12 It is difficult to resist the chance to point out that surprisal, as it is defined earlier, may represent the shock one experiences upon learning that the broker has told the truth.

13 Subject to the strong assumptions of no taxes or transaction costs. Although, in theory, one's capital may never be reduced to zero using the Kelly criterion, it is possible that capital may be reduced below a minimum required to place a bet. This is especially true for real estate.

14 This is the one asset, two valued payoff case.

(b) a positive payoff ($\frac{w}{l}$>1), or (c) a proper combination of (a) and/or (b) that produces a positive value for k. This constraint explains how the term "advantage gambling" came about.[15]

Rearranging and assuming the usual goal of maximizing one's wealth reveals that wealth gain $g(x)$, is a function of the size of the bet

$$g(x) = (1+xW)^{p} (1-xL)^{q} \qquad (6)$$

where:

p and q retain the meaning described above
x = size of the bet (portion of one's portfolio devoted to this investment)
W = win occurring in a single round
L = loss occurring in a single round

Taking the log of both sides

$$\mathrm{Log}[g(x)] = p\,\mathrm{Log}[1+Wx] + q\,\mathrm{Log}[1-Lx] \qquad (7)$$

Then setting the first derivative equal to zero

$$Log_x{}'[g(x)] = \frac{pW}{1+Wx} + \frac{qL}{Lx-1} = 0 \qquad (8)$$

Solving for x and letting q = 1-p produces our second interpretation, the optimal amount to bet for a single round

$$\text{Kelly criterion}_2 = x = \frac{pW-(1-p)L}{WL} \qquad (9)$$

A third approach uses a convenient estimate of log (1+x) to retrieve mean and variance parameters from actual data. The series approximation (close for small x)

$$Log_e\big(1+p(x)\big) \approx p(x) - \frac{p(x)^2}{2} \qquad (10)$$

leads to

$$\text{Kelly criterion}_3 = \frac{pW-qL}{p(W^2+\sigma_W)+q(L^2+\sigma_L)} \qquad (11)$$

where:

p and q have the meaning used above
W is the mean of the winning draws
L is the mean of the losing draws
σ_W is the variance of the winning draws
σ_L is the variance of the losing draws.

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15 Another variant is Blackjack where card counting confers an advantage on the player over the house.

With these three Kelly methods we can examine what happens with real data. Ignoring taxes and transaction costs, we create returns from the daily price change for Arbor Realty Trust (ABR) over an 80 month period between August 15, 2007 and February 15, 2014. Figure 1 shows the Kelly criterion for each method and the end result after investing $1,000 at the beginning of the period. The third version of Kelly, Eq. (11), using parameters of the data, produces the best final result with mid-range volatility.

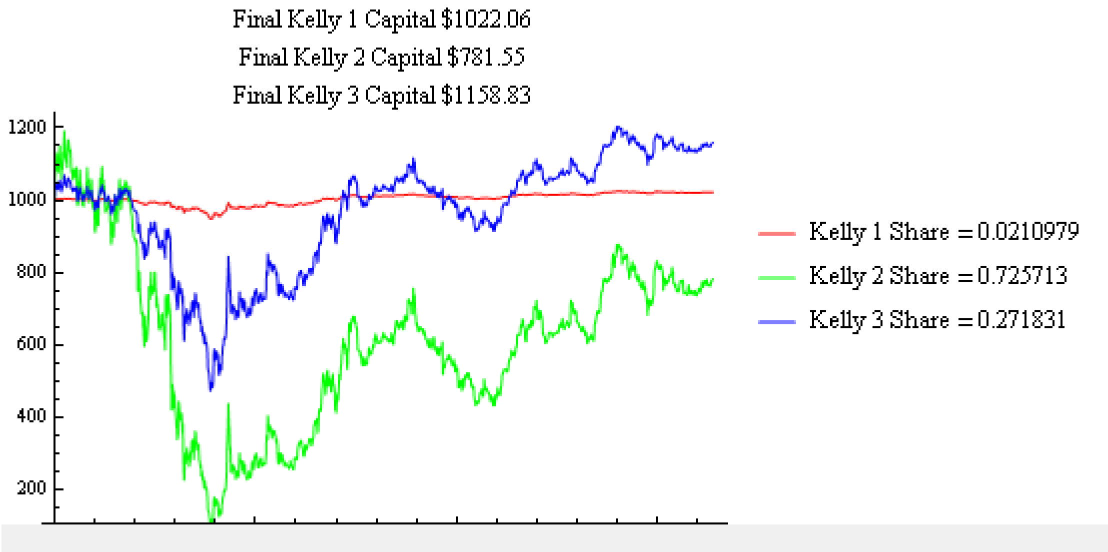

Figure 1 – Kelly criterion – three methods

V. What has this to do with real estate?

The short answer is "Not much." Except for the fact that the security used for the illustration in Figure 1 above has the word "Realty" in its name, the connection between the actual calculations associated with Kelly betting and real estate investing is practically non-existent. Kelly betting depends on a stationary process. There is much doubt that even the stock market is identically and independently distributed. Real estate is even less likely to be i.i.d. It is not at all certain that a real estate investment trust is anything like direct real estate investment, especially at the Tier II level.[16] Indeed, the only thing that recommends its study as a proxy is plentiful, current and free data. It is upon this convenience that academic real estate goes wrong, claiming to study real estate when in fact it is studying finance. Acknowledgement of this truth leads to understanding key differences between real estate and finance.

All of that said, the opening question at the beginning of this paper still begs for an answer. While information theory has scant direct bearing on real estate, neither does finance theory. Those who reject the notion that information theory is not connected to real estate veer close to the point of admitting that finance theory is

[16] Recall that Tier II, per Brown (2004), is made up of those properties in the market between the very small and the very large, preferred by private real estate investors.

equally unrelated. This leads to essentials of research which, on their own merit, make real estate different and should be respected. A random list might include:

1. Do not misapply a theory to a subject just because data is convenient. While information theory has been the subject of active research, its broad application fails to subsume real estate even, if not especially, through the quasi-familiar Kelly doctrine applied to finance.

2. In his conclusion, Ritchie (1986) exhorts us to imitate Shannon's method rather than borrow his results. Extending this to real estate requires the academy to leave finance models behind and embrace a long overdue standalone paradigm for the field.

3. If the stock market is the domain of the "fair" game, the progeny of regressing to the mean, Brownian Motion and all that classic randomness entails; private real estate is the nest of the Advantage Player engaged in a "favorable" game with empirical evidence for heavy *right* tails.[17] Miss-specifying the probability distribution has the effect of increasing entropy in academic research, essentially adding more heat than light.

4. Much of information theory depends on the Law of Large Numbers, especially large n. When the number of rounds tends toward infinity one may expect convergence to parameters, simply because as one reaches the limit of data one eventually includes all the alternatives, including all the extreme values.[18] Owning real estate has some of those properties in that it is a long term bet; but it lacks the final punch line as no human owner can expect to enjoy an infinite holding period.

5. Information wants to be free. One reason for the explosion of applications in the digital world is that very large numbers of collectable data exist to be analyzed. Real estate's determination, in the field and in the ivory tower, to keep the data close to its chest is misguided and counterproductive. Returning to #3 above, entropy is compounded as brokers capitalize on this antiquated practice and add noise to the system.

6. Recent information users have created value from information that is given away. Brynjolfsson and Joo Hee Oh (2012) found the Internet offered hundreds of billions of dollars of "free" goods. Indeed, their university has taken information openness to a new level. MIT now offers educational content of more than 2000 courses online at no charge. MIT is also the sponsor of The Billion Prices Project which uses high-frequency item-level data to create real time inflation indices in dozens of countries.

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[17] Brown (2004) postulated three reasons for this outcome: (a) fixed land supply; (b) market players extend holding periods to achieve a positive result; and (c) owners add labor, sometimes known as "sweat equity", which enhances (as well as corrupts) the return calculation.

[18] Which is why one description of infinity is that place where the least likely thing MUST happen.

7. Kelly betting is a risk management tool that prevents ruin. If real estate is a game of putting all your eggs in one basket and watching that basket, it seems useful to think of the Kelly criterion as a sort of governor on the overly acquisitive, the over-use of leverage and the importance of choosing the right location early in the game.

8. MacLean, Thorp and Ziemba (2011) observe "The Kelly and fractional Kelly rules, like all other rules, are never a sure way of winning for a finite sequence."[19] If little can be counted on in the short term, the risk averse investor is steered to the long term. Real estate investment practically demands this.

9. Devotees of the study of heavy tails or, as it has become known, Extreme Value Theory have come to appreciate a sort of "barbell" allocation in which assets are divided, not necessarily equally, between cash or cash equivalents and risky assets. Private real estate could be the sort of Kelly bet that fits the risky side of that equation.

10. There will be those compelled to compare the main players of information theory, entropy and mutual information, to the Boy Scout knife of real estate research, linear regression. This is flawed for a number of reasons. One is that the articles of information theory are only about probabilities and unconcerned with values assigned to outcomes. Another is that curve bending, as regression is uncharitably known in some circles, often infers non-existent causality. Just because you can get a matrix to invert does not mean that x *causes* y. In the setting of efficient coding, however, the knowledge of Y unambiguously improves what you know from that when you only knew X. The statistical dependency between X and Y is, in information theory, at once both more certain and more abstract.

VI. Conclusion

We don't know if Shannon and Mandelbrot ever met. Bell Labs and the IBM Watson Center enjoyed many a collaboration. We do know that these two made great contributions to the Information Age. Shannon brought us the probability of signal processing across a noisy channel. Mandelbrot showed us the math behind cyclical, non-periodic, noise along electrical lines. Each has a connection to the mathematics of tail behavior.[20] Separately or together, their theoretical work had spillover effects on social science that are still being debated.

The purpose of this paper, if indeed it has one, is to continue that debate in the context of the soul of real estate academics. In a world where the symbolic representation of standard deviation causes the majority of practitioners' eyes to glaze over, there are many opportunities to go awry when trying to describe the theory of an activity that appears in Nature more often as a trade than a profession. Real estate, at all levels, is an advantage play where the race goes to the most diligent

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19 P. 2 of the original paper; page 564 of the anthology

20 In real estate we use "Extreme Value Theory"; in information science the term is "Large Deviation Theory" but the outcome is the same, which is to model rare occurrences.

search for the ball the seller and his broker is hiding. It is not clear that this activity is a profession any more than discus throwing.

Information theory delivers one message loud and clear: If it is time to accept that the Internet is not a fad, it is time to recognize the abject folly in attempting to contain, mask, hide, fence, shield, conceal, secrete, stash or otherwise preserve data from public view. The effort is flawed in its intent, archaic in its practice, and doomed in its future. Change is required if real estate practice is to raise itself above hucksterism in the direction of a profession and if academic real estate is to share the stage with other sciences.

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